LITERATURE CITED

1. L. A. Dorfman, Hydrodynamic Resistance and Heat Emission for Rotating Bodies [in Russian], Fizmatgiz, Moscow (1960).
2. L. M. Zysina-Molozhen et al., Heat Transfer in Turbines [in Russian], Mashinostroenie, Moscow (1974).
3. I. T. Shvets and E. P. Dyban, Air Cooling of Gas-Turbine Elements [in Russian], Naukova Dumka, Kiev (1974).
4. A. V. Shcheglyaev, Steam Turbines [in Russian], GÉI, Moscow (1940).
5. Yu. E. Makhovko et al., Teploénergetika, No. 7 (1972).
6. B. L. Levchenko et al., Enhancement of Adjustability of K-100-90 Steam Turbines [in Russian], NIINFORMTYaZhMASh, Moscow (1973).
7. G. A. Shapiro et al., Élektricheskie Stantsii, No. 6 (1970).
8. E. Kamke, Handbook of Ordinary Differential Equations, Akad. Verl.-Ges. Geest und Portig, Leipzig (1959).

EFFICIENCY OF CONVECTIVE CIRCULAR FINS WITH A TRIANGULAR PROFILE

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We propose a graph for determining the efficiency factor of circular fins with a triangular profile, as well as a table of the principal parameters of such fins with minimum mass.

For circular fins with a triangular profile, of the kind shown in Fig. 1, the convective heat flux through the base is given by the formula

$$Q_0 = 2\pi r_1 l\alpha\vartheta_0(\varphi + 1)\eta = -2\pi r_1 \frac{\delta_0}{l}\lambda\vartheta_0\theta_0' , \tag{1}$$

where $\varphi = r_2/r_1$, θ_0'' is the dimensionless temperature gradient near the base of the fin, and η is the efficiency factor of the fin, which depends on the geometric shape of the fin and on the dimensionless parameter

$$\sigma = \frac{2\alpha l^2}{\lambda\delta_0} = 2\,\mathrm{Bi}\left(\frac{l}{\delta_0}\right)^2 . \tag{2}$$

The function $\eta(\sigma)$ is defined by the relations in Eq. (1):

$$\eta = -\frac{2\theta_0'}{(\varphi+1)\sigma} . \tag{3}$$

In order to determine η we need a solution of the differential equation of the temperature field, which ($\rho = r/l$; $\Lambda = \delta/\delta_0$; $\theta = \vartheta/\vartheta_0$), on the assumption that the heat flux is propagated only in the radial direction (one-dimensional problem), has the form

$$\Delta\frac{d}{d\rho}\left(\rho\frac{d\theta}{d\rho}\right) + \frac{d\Delta}{d\rho}\left(\rho\frac{d\theta}{d\rho}\right) - \sigma(\rho\theta) = 0. \tag{4}$$

For triangular and trapezoidal profiles the dimensionless thickness Δ of the fin can be expressed linearly in terms of ρ:

$$\Delta = \frac{\varphi}{\varphi - 1} - \rho. \tag{5}$$

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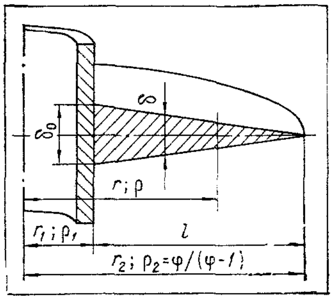

Fig. 1

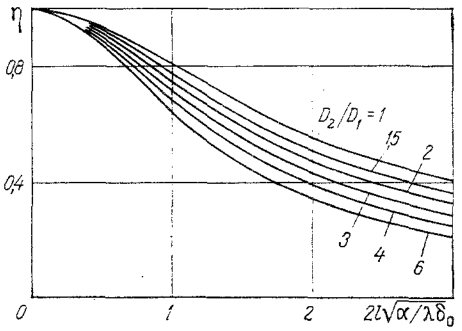

Fig. 2

Fig. 1. Diagram of a circular fin with triangular profile.

Fig. 2. Graph for determining the efficiency factor η as a function of the parameter $\sqrt{\sigma} = 2l\sqrt{\alpha/\lambda\delta_0}$.

After introducing the new variable $x = \sigma(\varphi/(\varphi - 1) - \rho)$ and the notation $\beta = (\varphi - 1)/\varphi\sigma$, we finally obtain the differential equation

$$x \frac{d^2\theta}{dx^2} + \left(1 - \frac{\beta x}{1 - \beta x}\right) \frac{d\theta}{dx} - \theta = 0 \tag{6}$$

with the boundary conditions (see [1])

$$x = 0; \ \frac{d\theta}{dx} = 0, \ x = \sigma; \ \theta = 1. \tag{7}$$

The special case of Eq. (6) when β = 0 corresponds to a fin with a large diameter and small height and has the general solution

$$\theta(x) = C_1 I_0 (2\sqrt{x}) + C_2 K_0 (2\sqrt{x}). \tag{8}$$

Fins with a triangular profile have no end face, so that the requirement of zero heat flux through the end face, stated in the form of the first boundary condition in (7), becomes trivial. In this case the constant of integration $C_2 = 0$ [1], and C_1 is found from the second condition in (7).

In the case $0 \leqslant \beta < 1$, Eq. (6) can be written

$$x \frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} - \theta = \frac{d\theta}{dx} \sum_{i=1}^{\infty} (\beta x)^i. \tag{9}$$

The solution of (9) for a triangular fin profile can be written in the form

$$\theta(x) = C_1 [I_0 (2\sqrt{x}) + P(x)], \tag{10}$$

where the first term is the solution of the homogeneous equation with a zero right-hand side, and the polynomial

$$P(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots \tag{11}$$

is regarded as the particular solution of (9) and is found by the method of successive approximations. In practice this leads to a step-by-step determination of the coefficients a_j according to the following scheme: using the values of the coefficients $a_0 = a_1 = 0$, a_2, a_3, ..., a_{j-1}, obtained from the previous cycles of the successive approximation, we determine $\theta_{j-1}(x)$ from (10). After differentiating $\theta_{j-1}(x)$ and replacing $I_0(2\sqrt{x})$ and $I_1(2\sqrt{x})$ with their series expansions, we add the terms on the right side of (9) with x^{j-1}. The next coefficient a_j is found from the equation

$$j^2 a_j - a_{j-1} = x^{-(j-1)} \left[\frac{d\theta}{dx} \sum_{i=1}^{\infty} (\beta x)^i \right]_{x^{j-1}}. \tag{12}$$

For example,

$$a_2 = \frac{1}{2!} \frac{\beta}{2}, \ a_3 = \frac{1}{3!} \left(\frac{\beta}{2} + \beta^2 \right), \ a_4 = \frac{1}{4!} \left(\frac{\beta}{4} + \frac{11}{8} \beta^2 + 3\beta^3 \right), \text{etc.} \tag{13}$$

TABLE 1. Optimal Characteristics of Circular Fins

φ	Fin with triangular profile			Fin with constant thickness		
	σ	η	v	σ	η	v
1	1,7147	0,5917	0,7037	2,0142	0,6267	1,0085
1,5	1,2287	0,6116	0,5312	1,5323	0,6361	0,8115
2	0,9882	0,6225	0,4143	1,2663	0,6424	0,6618
2,5	0,8464	0,6296	0,3312	1,1056	0,6455	0,5490
3	0,7516	0,6349	0,2708	0,9892	0,6488	0,4628
4	0,6338	0,6416	0,1911	0,8419	0,6523	0,3424
5	0,5628	0,6457	0,1426	0,7478	0,6549	0,2645
6	0,5142	0,6484	0,1109	0,6828	0,6567	0,2111
7	0,4779	0,6510	0,0889	0,6360	0,6568	0,1735
8	0,4491	0,6535	0,0730	0,6003	0,6571	0,1450

Thus, after finding C_1 from the second boundary condition of (7), we have

$$\theta(x)=\frac{I_0(2\sqrt{x})+P(x)}{I_0(2\sqrt{\sigma})+P(\sigma)}. \tag{14}$$

Since

$$\left(\frac{d\theta}{d\rho}\right)_{\rho=\frac{1}{\varphi-1}}=\left(\frac{d\theta}{dx}\right)_{x=\sigma}\left(\frac{dx}{d\rho}\right)_{\rho=\frac{1}{\varphi-1}}=-\sigma\frac{\sigma^{-\frac{1}{2}}I_1(2\sqrt{\sigma})+\left(\frac{dP}{dx}\right)_{x=\sigma}}{I_0(2\sqrt{\sigma})+P(\sigma)}, \tag{15}$$

we obtain from (3) the following formula for η:

$$\eta=\frac{2}{\varphi+1}\,\frac{\sigma^{-\frac{1}{2}}I_1(2\sqrt{\sigma})+\left(\frac{dP}{dx}\right)_{x=\sigma}}{I_0(2\sqrt{\sigma})+P(\sigma)}. \tag{16}$$

Formula (16) was used for constructing the graph shown in Fig. 2 for determining the efficiency factor η as a function of the parameter σ for various values of $\varphi = D_2/D_1$. Since analogous graphs for other variants of the design shape of the fin are given in [2, 3], for better comparability of the graphs we have replaced the parameter σ with $\bar{\sigma}$, defined on the basis of the average thickness of the fin $\bar{\delta} = {}^1/_2(\delta_0 + 0) = {}^1/_2\delta_0$:

$$\bar{\sigma}=\frac{2\alpha l^2}{\lambda\bar{\delta}}=\frac{4\alpha l^2}{\lambda\delta_0}, \tag{17}$$

and on the abscissa axis we have laid off values of $\sqrt{\bar{\sigma}}$.

The design choice of the fins based on Fig. 2 does not unambiguously answer the question concerning the optimal dimensions of the fins. It is possible to use different variants of the geometric dimensions giving us the same thermal efficiency. The choice of fins is often optimized by the condition that we must use a minimum amount of material (minimum mass) for constructing a fin with a given thermal efficiency. For fins made of a homogeneous material this problem reduces to the condition of minimum volume. Since for the limiting case $\varphi = 1 (r_1 \to \infty)$ the volume of a fin of finite dimensions approaches infinity, we shall try to find the minimum volume for a fin segment per meter of perimeter of the base:

$$V_1=V/2\pi r_1=\int_{r_1}^{r_2}\frac{r\delta}{r_1}\,dr=\delta_0 l(\varphi-1)\int_{\frac{1}{\varphi-1}}^{\frac{\varphi}{\varphi-1}}\Delta\rho\,d\rho=\min. \tag{18}$$

After finding Δ from formula (5), we can integrate (18), and after finding $\delta_0 l$ from (1), taking account of (3) and (5), we obtain

$$V_1=\frac{Q_1^3(\varphi+2)}{24\lambda\alpha^2\vartheta_0^3\sigma}\left(\frac{I_0(2\sqrt{\sigma})+P(\sigma)}{\sigma^{-\frac{1}{2}}I_1(2\sqrt{\sigma})+\left(\frac{dP}{dx}\right)_{x=\sigma}}\right)^3, \tag{19}$$

where $Q_1 = Q_0/2\pi r_1$ is the thermal effectiveness of the fin per meter of base. For a given value of φ the conditional minimum for V_1 is obtained when

$$\partial V_1/\partial \sigma = 0. \tag{20}$$

Equation (20) was solved for a number of values of φ. From the resulting values of σ we calculated the corresponding values of η by formula (16) and also the relative volumes of the fins of optimum design:

$$v = \frac{V_1}{V_0} = \frac{2}{3}\,\frac{\varphi + 2}{\sigma(\varphi + 1)^3\eta^3}, \tag{21}$$

where

$$V_0 = \frac{Q_1^3}{2\lambda\alpha^2\vartheta_0^3}. \tag{22}$$

The results of these calculations are shown in Table 1. Taking account of the fact that data of this kind have not yet been published for fins with constant thickness, we have calculated the data for these by a similar method and included them in the table. Using the table, we can determine the optimal values of l and δ_0 for circular fins by formulas (1) and (2), respectively.

NOTATION

a_j, coefficients of the polynomial (11); $Bi = \alpha\delta/\lambda$, Biot's similarity number; C_1, C_2, constants of integration; D_1, D_2, inner and outer diameters of the fin; I_0, K_0, modified Bessel functions; l, height of fin; Q, heat flux; r, radius; V, volume of the fin; v, dimensionless volume in (21); x, variable; α, heat-transfer coefficient; $\beta = (\varphi - 1)/\varphi\sigma$, coefficient; $\Delta = \delta/\delta_0$, dimensionless thickness of the fin; δ, thickness of the fin; η, efficiency factor (3); υ, excess temperature of the fin; $\theta = \vartheta/\vartheta$, dimensionless excess temperature; λ, thermal conductivity; $\rho = r/l$, dimensionless radius; σ, fin parameter in (2); $\varphi = D_2/D_1$, ratio of diameters.

LITERATURE CITED

1. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Énergiya, Moscow (1975).
2. Thermal Design of Boiler Installations (Normative Method) [in Russian], 2nd revised ed., Énergiya, Moscow (1973).
3. P. A. Andreev, D. I. Gremilov, and E. D. Fedorovich, Heat-Exchange Apparatus of Nuclear Power Installations [in Russian], Sudostroenie (1965).